Written Exam at the Department of Economics winter 2020-21

Financial Theory and Models

Final Exam

21 December 2020

(3½-hour open book exam)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

This exam question consists of 2 pages in total

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.

Be careful not to cheat at exams!
You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispelling from the exam. In most cases, the student is also expelled from the university for one semester.

Problem 1. Consider a stock in a binary two-period model with value 100 USD at time $t = 0$. We assume that the value in each node increases by 15 percent in the up-state and decreases with 5 percent in the down-state. The risk free interest rate $r = 5$ percent.

(i) Calculate the risk neutral probability distribution at maturity.

(ii) Calculate the arbitrage free price c of a European call option written on this stock with strike price $K = 100$ USD.

Problem 2. The police is observing speeding cars on a highway. We assume that the speeding cars arrive independently and at random to the checkpoint. The average number of speeding violations is 4 during each thirty minutes period.

(i) What is the distribution of the number of speeding cars observed during a thirty minutes period?

(ii) What is the probability that exactly 3 speeding cars are observed during a thirty minutes period?

(iii) Let Y_1 denote the stochastic variable that measures the time in hours from the moment that the observations at the checkpoint begin until the first (or more) speeding violations are observed. What is the distribution of Y_1.

Problem 3. Consider the Brownian motion $(B_t)_{t\geq 0}$ with its natural filtration

$$\mathcal{F}_t = \sigma\big(B_s \mid 0 \leq s \leq t\big).$$

(i) Show that $B_t - B_s$ is independent of $\mathcal{F}_s$ for $0 \leq s \leq t$.

(ii) Calculate the conditional expectation

$$E\big[(B_t - B_s)^2 \mid \mathcal{F}_s\big]$$

for $0 \leq s \leq t$.

Problem 4. A traded CO_2-permit has an expiry date $T > 0$ at which the value of the permit drops to zero. The value Y_t of the permit may therefore conveniently be modeled by setting

$$Y_t = \exp X_t\,,$$

where X_t is an Ito process given on differential form

$$dX_t = -\frac{b}{T-t}\,dt + \sigma\,dB_t\,,$$

B_t is the Brownian motion, and b, σ are positive constants.

(i) Integrate dX_t to obtain a formula for X_t .

(ii) Calculate the mean $E[Y_t]$.

(iii) Show that $E[Y_t] \to 0$ for $t \to T$.

(iv) Use Ito's lemma to write the process Y_t on differential form.